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No. 617

INVESTIGATION OF CERTAIN WING SHAPES WITH SECTIONS  
VARYING PROGRESSIVELY ALONG THE SPAN

By L. Arsandaux

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INVESTIGATION OF CERTAIN WING SHAPES WITH SECTIONS  
VARYING PROGRESSIVELY ALONG THE SPAN\*

By L. Arsandaux

Many engineers consider that the aerodynamic viewpoint is of little practical importance in the conception of a new airplane and that their attention should be given almost exclusively to constructional considerations. Many intentionally neglect the reading of aerodynamic treatises, considering the time thus employed as practically wasted. We consider it worth while, however, to call their attention to the present article. It will show them the possible effect of the form design not only on the performance of an airplane, but also on the stresses undergone in flight. Certain recent accidents, due to wing failure in flight, suggest the importance of such considerations. No systematic analysis of the forces acting on the wings has previously been undertaken. The method of designing set forth here would, if the author is right, produce both lighter and stronger wings.

The Editor.

This investigation has a double object:

- 1) The calculation of the general characteristics

( $C_m$  in particular) of certain wings with progressively varying sections;

- 2) The determination of data furnishing, in certain particular cases, some information on the actual distribution of

\*"Etude, calcul et determination de certaines formes d'ailes a profil évolutif le long de l'envergure." From L'Aéronautique, 1928: March, p. 99; April, p.121; May, p.167.

the external forces acting on a wing.

We shall try to show certain advantages belonging to the few wing types of variable section which we shall study and that, even if the general aerodynamic coefficients of these wings are not often clearly superior to those of certain wings of uniform section, the wings of variable section nevertheless have certain advantages over those of uniform section in the distribution of the attainable stresses.

The assumptions not being in strict accord with the reality, the numerical results necessarily have a slight margin of uncertainty. They enable, however, the comparison, from the viewpoint of economy of construction, of various practically realizable wings of progressively variable section.

Unfortunately, we were not able to make all the tests necessary for the complete justification of the proposed method. We shall therefore confine ourselves to the presentation of the results of two wind-tunnel tests of models to justify our calculations.

### I. Pressure Distribution on the More Common Wing Types

The designing of every new airplane type necessitates, in the first place, a knowledge of the aerodynamic polar of the desired wing. This is generally the only aerodynamic information accessible to the designer. This polar determines, for each position of the wing with respect to the direction of motion, the magnitude and direction, with respect to a reference

line connected with the wing, of the result of the local aero-dynamic pressures on each point of its surface. It does not, however, furnish any indication as to the distribution and magnitude of the elementary pressures on the surface of the wing. Unfortunately it is not possible to determine the distribution of the local pressures by wind-tunnel tests without going to great expense. For the lack of a better way, it is generally assumed, in the calculations, that the pressure distribution is uniform throughout the span and that it follows a uniform law along the chord regardless of the location of the section investigated with respect to the span. This is the implicit assumption in most of the regulations for strength tests. We shall show that, in a number of cases, this is far from correct.

1. Rectangular wings of uniform section.— The N.A.C.A. Technical Report No. 150, embodying the results of a systematic investigation, furnishes valuable information on this subject. The following general conclusions may be derived from it.

The pressure distribution along the span is far from being uniform, except at very small angles of attack (Fig. 1). At high lifts it follows a curve presenting several maxima and minima along the span, the greatest maxima being located near the wing tips.

The pressure distribution along the chord differs considerably from one section to the next. While the pressure diagrams of the sections situated near the median section resemble the

theoretical diagrams, those of the extreme sections exhibit very decided anomalies. On most of the models forming the subject of the N.A.C.A. Technical Report No. 150, a very perceptible negative pressure appeared on the upper side of the wing and toward the trailing edge of the outer portions, as soon as the utilizable angles of attack were reached. This phenomenon is common to rectangular wings of uniform section.

The relative importance of this anomaly can be rendered easily appreciable in the following manner. Let us assume that we have determined the pressures at every point and for every angle of attack of the wing under investigation. Let us then consider one of the halves of the wing as divided by the plane of symmetry, and let us determine, from the pressure diagrams, the center of pressure of this half for each angle of attack. If all the sections should behave in this way, or, which amounts to the same thing, if the distribution should be uniform throughout the span, as generally assumed in calculations, the positions of the centers of pressure for all the angles of attack should be in the plane of the section passing through the center of the surface of the half-wing under investigation. Moreover, this C.P. should, according to the theory, travel continuously toward the leading edge, when the angle of attack is increasing, while tending, as we shall see, toward a point situated at about 25% of the chord from the leading edge.

An analysis of the experimental results referred to above

leads to the conclusion that, contrary to the customary assumption, the C.P. of the half-wing, when the angle of attack increases, travels simultaneously along the chord and along the span, up to 10% of the half-span. It appears to be proportional to the thickness of the section and the camber of the mean chord. On the other hand, at large angles of attack, when the angle of attack increases, the C.P. moves toward the trailing edge of the wing, due to the appearance of marginal negative pressure.

It is important to note that, for all the rectangular wings of uniform section described in the N.A.C.A. Technical Report No. 150, the centers of pressure for the large angles of attack are generally located beyond the center of the surface of the half-wing, and conversely. Hence the pressures calculated on the assumption of uniform distribution are frequently overestimated for the ordinary angles of attack and underestimated for the case of flight at the extreme forward position of the C.P.

2. Tapering wings of uniform section.-- The anomalies noted for a rectangular wing of uniform section tend to disappear in proportion to the diminution of the ratio  $m = e''/e'$  of the chord at the tip to the chord at the middle, until  $m$  reaches a value of about 0.5 (Fig. 2).

- 1) The travel of the C.P. of the half-wing along the span becomes very small and the recession of the C.P. approaches zero at large angles of attack.

2) The pressure distribution along the span tends to approach uniformity.

When  $m$  becomes less than 0.5, the above-mentioned anomalies tend to reappear but, as regards the travel along the span, in the direction opposite to that on a rectangular wing.

It seems, therefore, that for a given angle of attack, the pressure distribution tends to become uniform along the span, when  $m$  reaches a value of about 0.5.

3. Tapered wings of variable thickness along the span and with corners rounded.— Three wings of this type, with an aspect ratio of about 6, were tested by the National Advisory Committee for Aeronautics, and form the subject of N.A.C.A. Report No. 229. An examination of the diagrams resulting from the tests leads to the conclusion that the pressure distribution along the chords of the different sections closely resembles that obtained in the median section of a rectangular wing of uniform section having the same profile as that of the given section and the same aspect ratio as that of the tapered wing, the C.P. of the half-wing varying very little laterally from the center of the corresponding plan form. Consequently, in the case of a tapered wing with a section decreasing in relative maximum thickness and of trapezoidal plan form such that  $m$  is practically 0.5, everything takes place as if, for every section, the flow were equivalent to a plane flow.

Let us now compare this conclusion with the theoretical result obtained by Prandtl, that when the distribution of the circulation (or lifting force per unit length) along the span is elliptical, the induced angle of attack is uniform for every section, regardless of its location.

Let us, moreover, impose the restriction that the sections of all the wings to be investigated shall have an angular adjustment with respect to one another, such that their position of zero lift shall correspond to the same position of the wing with respect to the direction of the air flow.

We may then assume that, taking into account a uniform induced angle of attack along the whole span, the flow over every section of an elliptical wing and, by extension, of a trapezoidal wing ( $m = 0.5$ ) satisfying these conditions, may be considered as a plane flow, regardless of the angle of attack. The purpose of these restrictions is to extend the law of elliptical lift distribution to all angles of attack.

On the basis of the hypotheses mentioned in the first part of this article, it is possible to assume in our calculations that, for wings satisfying the established program, everything takes place as if any element of the wing bounded by two neighboring planes parallel to the plane of symmetry would function in the same manner as the median element of a wing of constant section having the mean section of the element in question and the aspect ratio of the wing under investigation.

The immediate consequences of our manner of varying the wing section are:

1) To simplify the calculations;

2) To facilitate the centering, by reason of a property which we shall indicate further on;

3) Lastly, to present considerable practical interest.

Let us suppose, in fact, that all the sections of a wing are somehow adjusted, with respect to one another, to the zero-lift position of the wing. The general resultant of the lifts will then be zero, though some of the sections will have a positive and some a negative lift, so that in the case of a dive at the speed limit, local bending stresses are superposed on torsional stresses, thus producing supplementary stresses in the members. These stresses may be quite large and are very difficult to determine. At present no allowance is made for them in any static test.

It does not necessarily follow from these considerations that the wings which satisfy our program are the only important ones. It is possible that certain other wings with a different arrangement of the sections from that proposed may prove to be important if their general aerodynamic properties should permit of offsetting the increase in weight resulting from the increase in the local stresses undergone in flight, as compared with the local stresses on the proposed wings (See S.T.Aé. wing polars Nos. 8C and 9C). It will not be possible to arrive at any def-

inite conclusion on this point, so long as no systematic tests have been made for the purpose of finding whether these variations may be capable of improving the wing polar. The determination of the local stresses will still be very difficult and often unreliable, especially in the case of diving at the speed limit.

Nevertheless, as regards the above observations on diving flight, it seems reasonable for pursuit and racing planes (more often in the case of an airplane flying at a small value of  $C_Z$ ), to use only wings which satisfy the above-mentioned program, all the more because the reaction, on the polar, of the variation in the positions corresponding to zero lift would seem to be to increase the fineness considerably at small lifts.

The term "tapered wings," or, more often, "wings with progressive sections" will, in what follows, designate wings in which one or more of the characteristics (chord,  $C_{m_0}$ , relative thickness, or any other parameter susceptible of characterizing a section) vary in a continuous and decreasing manner from the plane of symmetry toward the tips.

To say that the positions of zero lift of the sections of a wing correspond to the same position of this wing with respect to the direction of flow, does not signify that the interinclination of the sections, as generally understood, is zero. This notion of interinclination has no meaning from the aerodynamic viewpoint, because it refers to a purely geometric relation be-

tween the chords of the sections and because the chord of a section ordinarily has no aerodynamic significance. In particular, if a wing consists of sections whose relative ordinates diminish in the same proportion from the median section toward the tips, the interinclination of the chords may be quite large, when their positions of zero lift correspond to one and the same position of the wing (Fig. 4).

## II. Definitions of Certain Characteristics of a Tapered Wing

Definition.-- The focus of a section is a point in the plane of the section such that the aerodynamic forces acting on the section are reduced, with respect to this point, to a force equal to the lift and to a constant moment regardless of the angle of attack. The theoretical demonstration of the existence of the focus is long and difficult. Moreover, it is valid only for an infinite aspect ratio. We shall confine ourselves to proving its existence on the basis of the results of experiments performed on wings of finite aspect ratio. We shall show that its position practically coincides with that of the theoretical focus of a wing of infinite aspect ratio.

Theorem.-- When the curve of the  $C_m$  values in terms of  $C_z$  is comparable to a straight line, the section admits a focus.

Let us consider, for example, a  $C_m$  curve defined in terms of  $C_z$ , as is the case in the S.T.Aé. diagrams. The equation for the  $C_m$  curve with respect to the leading edge (Fig. 5) may

be written in the form

$$C_{m_a} = C_{m_0} + f(C_z) C_z, \quad (1)$$

where  $f(C_z)$  designates the slope with respect to the  $C_z$  axis from the straight line joining the point A, representative of  $C_{m_0}$ , to the point B of the ordinate  $C_z$  on the  $C_m$  curve.

With respect to any point P, other than the leading edge, on designating by  $x$  the distance from the point P to the leading edge and by  $t$  the chord of the median section, we obtain

$$C_{m_n} = C_{m_0} + f(C_z) C_z + \frac{x}{t} C_z \quad (2)$$

If F denotes the focus, we shall have

$$C_{m_f} = C_{m_0} + f(C_z) C_z + \frac{d}{t} C_z = \text{constant} \quad (3)$$

where d denotes the distance from the focus to the leading edge.

However, according to the definition of focus,  $C_{m_f}$  is constant regardless of the angle of attack or of  $C_z$  which amounts to the same thing. The condition for the existence of the focus is then expressed by

$$C_z [f(C_z) + \frac{d}{t}] = \text{constant} \quad (4)$$

On referring to equation (3), we find that this constant must be zero, since  $C_m = C_{m_0}$  when  $C_z = 0$ , regardless of the point with respect to which the moments are taken. Hence, in order for the focus to exist, we must have

$$f(C_z) = \text{constant} = -\frac{d}{t} \quad (5)$$

or, in other words, the  $C_m$  curve must be comparable to a straight line and vice versa.

Consequently, in order to determine the focus of a section from experimental results, it is only necessary to draw through the origin (Fig. 6) a parallel to the  $C_m$  line and to read, at the point of intersection of this parallel with the scale of the centers of pressure, the position of the focus on the chord of the section. Experience shows that the position of the focus is practically independent of the aspect ratio, since the theoretical focus, determined on the assumption of an infinite aspect ratio, is situated, with respect to the experimental focus for customary aspect ratios, at a distance rarely attaining 2% of the chord of the section.

#### Consequences of the Existence of a Focus

The moment coefficient (with respect to the focus) of the aerodynamic stresses undergone by the section is constant and equal to  $C_{m_0}$ . This follows directly from formula (3).

The relative displacement of the C.P. (with respect to the corresponding elementary variation in lift) is proportional to  $C_{m_0}$  and inversely proportional to  $C_z^2$ . In fact, with respect to the focus, we have

$$C_{mf} = \text{constant} = C_{m_0},$$

regardless of the angle of attack. Nevertheless, at the angle of attack corresponding to  $C_z$ , if  $\lambda$  denotes the distance be-

tween the C.P. and the focus, we have

$$\lambda = \frac{C_m}{C_z} = \frac{\text{constant}}{C_z},$$

whence

$$\frac{d\lambda}{dC_z} = d \frac{C_m}{C_z} = - \frac{C_{m_0}}{C_z^2}.$$

If  $C_{m_0} = 0$  (symmetrical profile, for example), the resultant always passes through the focus.

Focus of a wing.— Let us consider a wing which satisfies the restrictions we have imposed and trace on its plan form (Fig. 7) the locus of the foci of the sections taken separately. The forces acting on an element of width  $dx$  in the direction of the span and of chord  $y$  are reduced, with respect to the focus  $f$  of the mean section, to a single force  $F$  equal to the lift of the element considered and to a constant moment, regardless of the angle of attack, according to the definition of the focus. Still, if we combine geometrically all the elementary forces applied to each of the elementary foci  $f$  for a given angle of attack, we shall obtain a resultant  $R$ , applied at a point  $G$  located in the plane of symmetry of the wing and having the magnitude of the total lift of the wing. Each of the elementary forces  $F$  being proportional to the angle of attack measured from the position of zero lift, which is assumed to be the same for all the sections, the position of  $G$  is

therefore fixed regardless of the angle of attack. By analogy with the definition of the focus of a section, we shall call the point G the focus of the wing. All that has been said regarding the focus of a section is applicable to the focus of a wing. Consequently, we have the following definition.

The focus of a wing is a point in the plane of symmetry of the wing where the aerodynamic pressures acting on the wing are reduced, with respect to this point, to a force equal to the total lift of the wing at the given angle of attack and to a constant moment, regardless of the angle of attack. Hence the sum of all the forces acting on the wing, with respect to the focus G, will be equivalent to:

- 1) A force R, the geometric resultant of the elementary lifts on each section, or

$$R = 2 \int_0^{b'} C_z y \frac{aV^2}{2g} dx \quad (\text{total lift})$$

$b'$  denoting the half-span.

- 2) A moment  $M_o$ , which is constant regardless of the angle of attack and equal to the resultant of the elementary moments for the different sections, or

$$M_o = 2 \int_0^{b'} C_{m_o} y^2 \frac{aV^2}{2g} dx \quad (\text{total moment})$$

The mean relative thickness of a tapered wing is the quantity

$$E_m = \frac{\int \epsilon y dx}{S}$$

the symbol  $\int$  being extended to the whole span and  $\epsilon$  denoting the relative thickness of the section situated at the distance  $x$  from the plane of symmetry.

The resultant coefficient  $C_z$  for a wing of infinite span can be put in the form

$$C_{z\infty} = K \sin i,$$

where  $i$  denotes the angle of attack and  $K$  a particular constant for the wing - being, moreover, a practically linear function of the maximum relative thickness. Since the practically utilizable angles of attack are small, we can write

$$C_{z\infty} = K i,$$

$i$  being expressed in radians.

On the other hand, it is known that the value of  $C_z$  for a wing of finite span bears a constant ratio to the corresponding  $C_z$  of a wing of infinite span at the same angle of attack, this constant ratio  $k$  depending only on the aspect ratio and the wing section. Hence, for a finite aspect ratio, we can write

$$C_z = K i k.$$

The resultant  $C_z$  of a tapered wing will then have the value

$$C_z = i k \int \frac{K y dx}{S} \quad (6)$$

which can be written in the form

$$C_z = i k K'. \quad (6')$$

The value of the resultant  $C_z$  for a tapered wing, such as we have defined, is always less, for a given angle measured from the position of zero lift, than the value of the same coefficient for a rectangular wing of uniform section having the profile of the tapered wing with the mean maximum relative thickness and the aspect ratio of the tapered wing.

### III. Determination of Wings Satisfying a Given Linear Relation $C_m = f(C_z)$

When the curve  $C_m = f(C_z)$  is a straight line, it is completely determined by its slope with respect to one of the coordinate axes and one of its points. In other words, we know the straight line representing the  $C_m$  of a tapered wing satisfying the stipulated conditions, when we know how to determine the value of its total  $C_{m_0}$  and its focus  $G$ .

1. Determination of the total  $C_m$  for a tapered wing.— In aerodynamic laboratories it is customary to express the results of the measurements of pitching or torsional moments by using the nondimensional coefficient  $C_{m_0}$  as determined from the well-known formula

$$M = C_{m_0} S l \frac{aV^2}{2g} \quad (a)$$

$M$  being the moment measured in the wind tunnel;  $S$ , the wing area;  $l$ , a characteristic dimension of the wing, generally taken in the plane of symmetry (uniform chord for a rectangular

wing, chord of median section for a wing of any plan form);  
 $\rho$ , specific weight of air under the conditions of the experiment;  
 $V$ , velocity of air stream;  $g$ , acceleration due to gravity.

At the position of zero lift

$$M_O = C_{m_O} S l \frac{\rho V^2}{2 g} . \quad (b)$$

In the particular case of a rectangular wing of uniform section,  $l$  being constant throughout the span, the value of  $C_{m_O}$ , as determined in the laboratory with the aid of formula (b), must be identical with that of the same coefficient as determined by calculation from the section or by any other method. This method has been successfully employed in various laboratories, the experimental results seeming to be in good accord with theoretical hypotheses, when the camber of the section is not over 5% of the chord.

Under these conditions it is possible to calculate the torsional moment at the position of zero lift for a wing with chord and section varying progressively along the span, knowing its plan form and theoretical characteristics as calculated for each section. In fact, since the value of  $C_{m_O}$  is independent of the aspect ratio, we can apply the formula (b) to any element of this wing bounded by two neighboring planes parallel to the plane of symmetry of the wing. A summation, throughout the span, of the torsional moments undergone by each element will enable us to determine the total torsional moment and conse-

quently the total  $C_{m_0}$  or wing resultant for any chord, the chord of the mean section being generally used.

General case.-- The elementary moment at the angle of zero lift, due to a rectangular surface element of width  $dx$  and chord  $y$ , determined as indicated above has, by reason of formula (b) and the previously adopted hypotheses, the value

$$d M_0 = \frac{aV^2}{2g} C_{m_0} y^2 dx, \quad (7)$$

$C_{m_0}$  being the moment coefficient of the section situated in the plane XX parallel to the plane of symmetry. Since the formulas for the variation of  $C_{m_0}$  and  $y$  along the span are

$$C_{m_0} = F(x), \quad y = \Phi(x),$$

formula (7) can be expressed in the form

$$d M_0 = \frac{aV^2}{2g} F(X) [\Phi(x)]^2 dx, \quad (7')$$

$X$  being the value of  $x$  for the section XX. The total torsional moment in the section XX is determined by the formula

$$M_{0XX} = - \frac{aV^2}{2g} \int_a^X f(x) [\Phi(x)]^2 dx, \quad (8)$$

the integral being extended to the portion of the wing included between its tip and the section XX. The torsional moment in the median section is expressed by

$$M_{0YY} = + \frac{aV^2}{2g} \int_0^b F(x) [\Phi(x)]^2 dx, \quad (9)$$

which may be simplified to

$$M_{0YY} = \frac{av^2}{2g} K'' S l, \quad (9')$$

$K''$  designating a coefficient peculiar to the wing under consideration and a function of the values of  $C_{m_0}$  for each section (particularly in the plane of symmetry and at the wing tip), a function resulting from the law of progressive variation adopted for  $C_{m_0}$  along the span, and  $l$  designating in general the chord of the median section. It is obvious that the value calculated for  $K''$  must correspond to the value of the total  $C_{m_0}$  as determined in the laboratory.

We give below the results of this calculation in certain important cases which we will examine more closely further on.

#### N o t a t i o n

$C$ ,  $C_{m_0}$  of the median section;

$c$ ,  $C_{m_0}$  of the tip section;

$l$ , chord of median section, in the case of an elliptical wing or, more generally, of a wing with progressively varying sections;

$l'$ , chord of median section of a tapering wing;

$l''$ , chord of tip section of a tapering wing;

$m = l''/l'$  for a tapering wing;

$n = c/C$  for a tapering wing:

The following formulas apply only when the lift distribution is practically elliptical along the span or, in the case of a tapered wing, when the value of  $m$  differs but little

from 0.5. The less peculiarities (sharp angles, sudden variations in the radii of curvature, etc.) there are in the plan form of the wing, the more accurate these formulas are.

1) Wing with elliptical distribution and constant  $C_{m_0}$  (constant or variable section):

$$C = c,$$

$$M = \frac{2}{3} \frac{4}{\pi} \frac{aV^2}{2g} S C l = 0.850 \times \frac{aV^2}{2g} S C l,$$

$$K'' = 0.850 C$$

2) Wing with elliptical distribution and elliptical evolution of  $C_{m_0}$  (variable section).

(A)

$$c = o,$$

$$M = \frac{3}{4} \frac{aV^2}{2g} S C l = 0.750 \times \frac{aV^2}{2g} S C l,$$

$$K'' = 0.750 C$$

(B)

$$c \neq o,$$

$$M = [0.750 (C - c) + c] \frac{aV^2}{2g} S l,$$

$$K'' = 0.750 (C - c) + c.$$

3) Wing with elliptical distribution and parabolic evolution of  $C_{m_0}$  (variable section).

(A)

$$c = o,$$

$$M = \frac{8}{15} \frac{4}{\pi} \frac{aV^2}{2g} S C l = 0.678 \times \frac{aV^2}{2g} S C l,$$

$$K'' = 0.678 C.$$

(B)

$$c \neq o,$$

$$M = [0.678 (C - o) + c] \frac{aV^2}{2g} S l,$$

$$K'' = 0.678 (C - o) + c.$$

- 4) Tapered wing with constant  $C_{m_0}$  (constant or variable section):

$$c = o,$$

$$M = \frac{2}{3} \times \frac{m^2 + m + 1}{m + 1} \frac{aV^2}{2g} S l,$$

$$K'' = \frac{2 (m^2 + m + 1)}{3 (m + 1)}.$$

- 5) Tapered wing with linear variation of  $C_{m_0}$  (variable section):

$$M = \frac{(1 + 3n) m^2 + 2 (1 + n) m + (3 + n)}{6 (m + 1)} \frac{aV^2}{2g} S C l,$$

$$K'' = \frac{[(1 + 3n) m^2 + 2 (1 + n) m + (3 + n)]}{6 (m + 1)}.$$

In order to obtain wings with linear variation of  $C_{m_0}$ , it is not enough to determine graphically the root section and the tip section, for example, and then the intervening sections by any rectilinear generation whatsoever. There is no reason why, under these conditions, the laws of variation of the aerodynamic

characteristics should be linear. In particular, any rectilinear generation of the intermediate sections of a tapered wing sometimes yields sections whose  $C_{m_0}$ 's are clearly superior to those of the two sections serving as directrices in the generating system.

2. Position of the focus of a tapered wing.— It follows from the above that the focus of a wing is the point of application of the resultant of the elementary lifts applied to the focus of the section involved.

Since there is a focus in the cases under consideration, it will coincide with the point of application corresponding to any angle of incidence of the resultant of the elementary lifts due to each surface element and applied to the elementary focus of the mean section corresponding to this element.

a) Magnitude of the resultant.— Let us consider the surface element previously defined and situated in a section XX' of abscissa X. Let  $dx$  denote its width along the span and  $y$  its chord. The wing being set at an angle  $i$  with respect to its position of zero lift, and the lift coefficient being proportional to this angle, we can write

$$C_{z_1} = K i k. \quad (6)$$

$K$  being a constant, we deduce

$$d R = k \frac{aV^2}{2g} K i y dx \quad (\text{elementary lift}).$$

Integration yields

$$R = k \frac{aV^2}{2g} i \int_{-b'}^{+b'} K i y dx = k \frac{aV^2}{2g} i \int_{-b'}^{+b'} K y dx \quad (10)$$

The value of  $K$  depends on the aspect ratio of the wing and the shape of the elementary section, and particularly on its relative thickness. Hence, in order to calculate the magnitude of the general resultant of the lifts  $R$  for any given angle of attack, we must know the two functions

$$K = g(x) \quad \text{and} \quad y = \Phi(x),$$

and write definitively,

$$R = k \frac{aV^2}{2g} i \int_{-b'}^{+b'} g(x) \Phi(x) dx \quad (10')$$

In the case of theoretically determined sections,  $K$  is obtained directly from the tracings of the sections.

b) Point of application of resultant.— If  $ff$  is the locus line of the foci of the sections of a wing and  $FF$  an axis passing through the ends of this line limited to the span, then  $FF$  is necessarily parallel to the span. Let  $z = h(x)$ , the analytical expression for the curve  $ff$  with respect to the axes  $FF$  and  $YY$ . If  $D$  is the distance between the wing focus  $G$  and the axis  $FF$ , we shall have, according to what precedes,

$$D \times R = k \frac{aV^2}{2g} i \int_{-b'}^{+b'} g(x) \Phi(x) h(x) dx,$$

whence

$$D = \frac{\int_{-b'}^{+b'} g(x) \Phi(x) h(x) dx}{\int_{-b'}^{+b'} g(x) \Phi(x) dx}. \quad (11)$$

## Examples

1. Tapered wing with uniform section.— The characteristic functions of such a wing are

$$(A) \quad C_{m_0} = \text{constant}$$

$$(B) \quad y = l' - (l' - l'') \frac{x}{b'}$$

$$(C) \quad K = \text{constant}$$

$$(D) \quad z = d \left(1 - \frac{x}{b'}\right)$$

denoting the distance from the axis FF' to the particular focus of the section situated in the median plane. Formula (11) yields the following result

$$D = \frac{m + 2}{3(m + 1)} d.$$

For a tapered wing such that  $m = \frac{2}{3}$ , we obtain

$$D = \frac{8}{15} d,$$

and for

$$m = \frac{1}{2},$$

we obtain

$$D = \frac{5}{9} d.$$

2. Wing of uniform section enabling an elliptical distribution of the chords and an ellipse for the locus line of the foci.— The characteristic functions of this wing are:

(A)  $C_{m_0} = \text{constant}$

(B)  $y = l \sqrt{1 - \frac{x^2}{b^2}}$

(C)  $K = \text{constant}$

(D)  $z = d \sqrt{1 - \frac{x^2}{b^2}}$

Formula (11) yields the simple result

$$D = \frac{2}{3} \times \frac{4}{\pi} d = 0.850 \times d.$$

Three wings of this type, of aspect ratio 5 and section G 430, were tested in the Saint Cyr laboratory. The calculated positions of the foci of these wings and the corresponding positions derived from the test results are compared below.

	Straight leading edge	Symmetrical	Straight leading edge
D' calculated, %	21.8	29.3	36.75
D' experimental, %	20.0	27.1	35.00

D' here represents the distance between the focus of the wing and the leading edge of the median section with respect to the chord of the section.

These figures show that we can obtain by calculation a close approximation of the position of the focus of a tapered wing, despite the relatively small aspect ratio of the models tested and the high camber of the G 430 section. On the other hand, it is interesting to note that, in the three above cases, the ex-

perimental focus is always about 2% of the median chord in front of the theoretical focus.

3. Wing of variable section, elliptical lift distribution and elliptical evolution of the  $C_{m_0}$ 's, enabling an ellipse for the locus line of the foci and for which  $c = 0$ . In this case the value of the function  $C$  may vary along the span. However, since it varies but little, it may generally be assumed in the calculations that it is constant and has the value

$$K_1 = \int_0^{b'} K \frac{y}{S} dx$$

$S$  being the total wing area.

The characteristic functions of this wing then become

$$(A) \quad C_{m_0} = C \sqrt{1 - \frac{x^2}{b'^2}}$$

$$(B) \quad y = l \sqrt{1 - \frac{x^2}{b'^2}}$$

$$(C) \quad K = \text{constant} = K_1,$$

$$(D) \quad z = d \sqrt{1 - \frac{x^2}{b'^2}}$$

Since the position of the focus is independent of the function  $A$ , the value of  $D$  is the same as in the preceding case, namely,

$$D = 0.850 d.$$

Thus we see that, in passing from the uniform distribution of the  $C_{m_0}$ 's to the elliptical distribution along the span without changing the plan form or the distribution of the maximum

relative thicknesses of the sections along the span, the position of  $G$  is not changed, provided  $K$  is constant. It varies but little, even when  $K$  varies progressively. We shall see farther on, that the effect of this modification is to reduce the torsional moment at every point, provided the total value of  $C_{m_0}$  is the same in both cases.

3. Practical significance of the focus.— The moment of the resultant of the forces acting on a wing being constant with respect to the wing focus, if we place the C.G. of the airplane on the vertical line passing through the wing focus assumed to be in the position of normal flight, and if we assume, moreover, that the moment produced by the tail is proportional to the angle of attack, the total moment to which the airplane is subjected will be proportional to the angular difference between its actual position and its position in normal flight.

If we combine all the external forces acting on the airplane with respect to the focus, and not with respect to the C.G., as is commonly done, it is possible to develop, in a simple mathematical form, equations for determining the static stability of the airplane. We may therefore conclude that the position of the focus determines the method of centering the wing on the airplane.

## Summary

1) A tapered wing corresponding to our program is characterized, from the aerodynamic viewpoint, by four functions:

$$(A) \quad C_{m_0} = F(x),$$

$$(B) \quad y = \Phi(x),$$

$$(C) \quad K = g(x),$$

$$(D) \quad z = h(x).$$

2) The value of  $C_{m_0}$  depends only on A and B.

3) The method of centering to apply to the wing (with respect to its median chord) depends on B and D and to a slight degree on C, but not at all on A.

Present methods of designing wing sections\* enable us to obtain series of sections satisfying the functions A and C, and fixed in advance according to the law of evolution of the torsional stress along the span, as well as the value of the total  $C_{m_0}$  it is desired to obtain. Only the functions B and D remain to be determined, in order to obtain the best centering for the wing. Consequently, the investigation of the centering and static stability of an airplane provided with such a wing can be entirely made without recourse to tests with models.

4. Practical application of these considerations to the design of a wing.— There are two feasible cases:

1) Determination of a wing whose plan form is not prescribed.

The conditions of utilization of the wing generally fix

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\*L'Aéronautique, December, 1927, and January, 1928.

the position of the C.G. with respect to the chord of the median section. By making the focus coincide with this position, we can deduce, by the foregoing formulas, the position of the particular focus of the median section, account being taken of the geometrical nature of the focus line adopted in advance for economic reasons.

The form of the different wing sections being defined on the basis of the median section and allowance made for the evolution of the different characteristics, it will only be necessary, in order to obtain the shape of the wing, to put all the sections in their respective places along the span (as determined by the value of  $x$  proper for each section, a value which, moreover, with the evolutive laws, determines the magnitude of their different parameters) by making the individual foci coincide with the locus line of the foci and their position of zero lift with the zero-lift position of the median section. The shape of the wing is thus completely determined.

### 2) Determination of a wing whose plan form is prescribed.

Constructional considerations may sometimes influence a designer to adopt a plan form whose focus line cannot be represented by a simple analytical expression. The wing focus can then be determined either by measuring the values of  $x$  on the drawing, or by adopting an expression of the form

$$z = A + B x^2 + C x^4 + D x^6 + \dots$$

for the locus of the foci with respect to the axes FF and YY.

#### IV. Pressure Distribution Along the Span

In Chapter I it was shown that, if we confine ourselves to wings such that (1) the pressure distribution along the span is practically elliptical, and (2) the positions of zero lift of all the constituent sections correspond to one and the same position of the wing with respect to the direction of flow, it is permissible to assume that (allowance being made for a constant induced angle for all the sections) everything takes place, from the viewpoint of pressure distribution, as if the flow were uniplanar throughout the span.

The immediate result is that:

1) The total pressure on an elementary section of width  $dx$ , taken at any point of the span of a practically elliptical tapered wing, can be represented, for a given velocity, by an expression of the form  $dR = K i y$ ;

2) The diagram of the pressures along the chord of any section will be the same as that obtained in the median section of a rectangular wing of uniform section, like the one considered, and having the same aspect ratio as the tapered wing under investigation.

On the basis of these results, it is possible for us to determine and compare the pressure distribution on the different wing types whose general characteristics were investigated in Chapter III. We shall study successively:

- 1) The distribution of the bending stresses;
- 2) The distribution of the torsional stresses at the position of zero lift.

The representative diagrams of these distributions are obtained by the application of the following formulas:

$$\begin{aligned} M &= \frac{av^2}{2g} k i \int_{b'}^X K(x - x) y dx \\ &= \frac{av^2}{2g} k i \int_{b'}^X g(x) \Phi(x) (x - x) dx \quad (\text{flexure}) \\ M_0 &= - \frac{av^2}{2g} \int_{b'}^X f(x) \Phi(x)^2 dx \quad (\text{torsion at zero lift}). \end{aligned}$$

The principal object of this investigation being to determine the advantages peculiar to the different wing types, rather than to expound in detail the calculations involved in the application of these two formulas, we shall confine ourselves to presenting, in a simple form, the comparative diagrams which represent them.

1. Comparative diagram of the bending moments.— This diagram (Fig. 8) makes it possible to compare the bending moments in sections similarly situated along the span of a rectangular wing and of an elliptical wing of uniform section, having the same profile, the same aspect ratio and the same area. We have plotted, as abscissas, the distances of the sections measured along the span from the plane of symmetry and, as ordinates, the values of

$$C = \frac{M}{C_Z l b'^2} \frac{1}{\frac{av^2}{2g}}$$

$l$  denoting the chord of the median section of the elliptical wing and  $b'$  the half-span common to both wings. This diagram is valid regardless of the angle of attack, provided  $C_z$  is below the value corresponding to the bending point of the  $C_z$  curve in terms of the angle of attack. It shows that the adoption of the elliptical plan form entails a reduction of 15% in the bending moment at the root, as compared with the rectangular wing. This relative reduction increases moreover, as the wing tip is approached.

If allowance is made for the fact that the median section of the elliptical wing is 1273 times that of the rectangular wing, it is found that the stresses in the spar flanges are reduced one-third. This relative reduction of the stresses increases, moreover, as the wing tip is approached. If the relative thickness of the elliptical wing decreased toward the tip, the  $C$  curve would be practically the same as the curve for an elliptical wing of constant section.

2. Comparative diagram of the torsional moments.— We have compared (Fig. 9) the torsional moments along the span for the following wing types having the same aspect ratio, the same area and the same over-all pitching moment:

- 1) Rectangular wing of uniform section (curve A);
- 2) Elliptical wing of uniform section (curve B);
- 3) Elliptical wing with elliptical variation of  $C_{m_0}$  (curve C);
- 4) Elliptical wing with parabolic variation of  $C_{m_0}$  (curve D).

We have plotted, as abscissas, the distances of the sections measured along the span from the plane of symmetry and, as ordinates, the values of

$$C' = \frac{m}{M} = \frac{m}{\frac{\alpha V^2}{2g} K'' S l},$$

$M$  being the total pitching moment of the wing and  $m$  the torsional moment in any section. It follows from the above that the value of the denominator is the same in all the four cases investigated. This diagram shows that the diminution of the torsional moment in a tapered wing is due to two distinct causes:

- 1) Variation in the chord distribution;
- 2) Variation in the  $C_{m_0}$  distribution;

It is obvious, for example, that at a distance from the plane of symmetry equal to half of the half-span, the passing from the rectangular to the elliptical form entails a reduction of 37% of the torsional moment at this point. In passing from the constant-section form to that with a parabolic evolution of  $C_{m_0}$ , there is a further reduction of 20%, making a total reduction of 57% as compared with the moment of the rectangular wing.

If, moreover, allowance is made for the fact that in this section the chord is 1.1 times that of the rectangular wing, it is found that the torsional fatigue undergoes a still greater reduction of as much as 65 to 70%. This reduction increases toward the wing tip.

For the reasons indicated in Chapter I, curve A gives only

a rough idea of the real stresses, while curves B, C, D more clearly approximate the reality.

These diagrams do not include the curves for tapered wings, which closely resemble the curves for elliptical wings, when allowance is made for the prescribed restrictions.

It might be easily shown that the form of a wing could not be derived from an arbitrary law of evolution for  $C_{m_0}$ , but from the law of variation of the desired torsional moment.

The production of wings of this nature requires great precision in tracing the profiles, since a relatively small error in the value of a parameter may cause a modification in the shape of the profile, which though often very slight is such a nature as to modify considerably the value of its  $C_{m_0}$ .

Thus far we have dealt only with the determination of the torsional moments at the position of zero lift, this being the most important question for the present. The investigation of the torsional moments for attitudes other than that of zero lift is much more laborious. These moments depend:

- 1) On four characteristic wing functions;
- 2) On the constructional system.

This question will, moreover, be the subject of another investigation.

It may be stated now, however, that for a wing of current design, with ribs and two spars, the torsional stresses at the same speed are generally less than the torsional stresses in the

position of zero lift, when  $C_Z$  is positive, and vice versa.

### V. Experimental Verification

#### Analysis of Results of Tests with Wing Model No. 2B

The plan form of this wing (Fig. 10) consists of two surfaces with practically elliptical distribution along their chords, separated by a central portion of uniform section. Its geometrical characteristics are: span, 1.2 m (3.94 ft.); median chord,  $l = 23$  cm (9.05 in.); maximum camber of median section, 20.6%; mean camber, 17.7%; total area,  $22.68 \text{ dm}^2$  (351.5 sq.in.); area of the two elliptical portions,  $19.92 \text{ dm}^2$  (308.8 sq.in.).

The  $C_{m_0}$  of its median section (Fig. 11), calculated from the tracing of this section, is 5.225. This wing, after deduction of the median part, admits the following characteristic functions:

$$(A) \quad C_{m_0} = C \sqrt{1 - \frac{x^2}{b^2}}$$

$$(B) \quad y = l \sqrt{1 - \frac{x^2}{b^2}}$$

$$(C) \quad K = 6.28 + 1.08 \sqrt{1 - \frac{x^2}{b^2}}$$

$$(D) \quad z = h(x).$$

The analytical expression of function D is not given. The plan form was determined on the basis of the following conditions: straight leading edge; straight upper flange of front spar; rounding of the wing tip, without changing the shape of the sec-

tions or their relative angle of attack, in such manner that the aileron axis comes entirely within the wing. Moreover, this wing satisfies the hypotheses set forth in Chapter I.

1. Calculation of the over-all  $C_{m_0}$ . - In calculating the over-all moment of the wing in the position of zero lift, we have the moment due to the elliptical portion

$$M_1 = \frac{aV^2}{2g} \times 0.750 \times C \times 0.1992 \times 0.230,$$

and the moment due to the central portion,

$$M_2 = \frac{aV^2}{2g} C \times 0.0276 \times 0.230,$$

making the total moment

$$M_t = \frac{aV^2}{2g} C (0.1992 \times 0.750 + 0.0276) \times 0.230.$$

Dividing the latter quantity by  $\frac{aV^2}{2g} S l$ , we get

$$C_{m_0} (\text{over-all}) = 4.06.$$

On comparing this figure with the result obtained with the model in the wind tunnel, it is found that the discrepancy is of the order of magnitude of the experimental errors.

2. Determining the position of the focus. - The locus line of each section was traced on the drawing of the plan form of the wing. The foci were determined graphically on the diagrams of the sections. The plan form of the locus of the foci being difficult to define analytically with sufficient precision, we measured the values of  $z$  at intervals of 3 cm (1.18 in.) throughout the span and calculated the expression

$$D = \frac{\sum K_y z \Delta x}{\sum K_y \Delta x}, \quad \Delta x = 3 \text{ cm.}$$

The effect of the aspect ratio on the coefficient  $K$  is eliminated in this formula. We may, however, introduce directly the values of  $K$  for infinite span, i.e., the values obtained from the diagrams. We thus obtain  $D = 58.8 \text{ mm (2.31 in.)}$ . Measuring off this distance on the axis of symmetry of the plan of the model in full scale starting from the axis FF, the focus G is found to be 55.2 mm (2.17 in.) or at 24% of the chord from the leading edge. This is the figure derived from the experimental results by the method already indicated. In short, the  $C_{m_0}$  curve, as calculated for the wing No. 2B, practically coincides with the corresponding experimental curve.

3. Pressure distribution along the chord.— It seemed to be of interest to add to these results the theoretical diagrams of the pressure distribution along the chords of sections No. 0 and No. 4 for angles of attack increasing by  $5^\circ$  at a time, starting from the position of zero lift, and for infinite aspect ratio (Figs. 12-13). According to the hypotheses adopted at the beginning of this investigation, these diagrams are the same for finite aspect ratios, excepting the angles of attack. It will only be necessary, however, in these systems of curves to modify the indication of the angle of attack for each curve, in order to obtain the corresponding diagrams for the wing No. 2B. These angles of attack are indicated in parentheses on each

curve beside the angle of attack for infinite aspect ratio. In Figures 12 and 13 the zero angle of attack corresponds to zero lift. The pressure at any point is given by the formula

$$p = k \times \frac{av^2}{2g}$$

Hence, at the angle of attack of  $+7.35^\circ$ , the pressure at the point A, situated on the upper surface at 25% of the chord from the leading edge is

$$p = 1.44 \times \frac{1600}{16} = 144 \text{ kg/m}^2$$

for a speed of 40 m/s.

On the basis of the results obtained by Toussaint and Carafoli, as published in L'Aérophile, April, 1927, these theoretical diagrams are found to approach the experimental diagrams closely enough, notably in the usual range of flying angles and especially in the neighborhood of zero lift, to justify their use in drag calculations. They can therefore aid in determining the order of magnitude of the forces acting on the ailerons and their hinges at different angles of attack. They would, e.g., furnish an excellent basis for calculating these forces in diving flight or in flight at maximum speed. These diagrams can also be advantageously used in calculating the ribs and determining the cases in which the stresses produced by the static tests are smaller than those arising in actual flight.

The angle of zero lift, measured from the tangent to the lower side of the median profile has the experimental value

$6.75^\circ$  instead of the theoretical value  $6.5^\circ$  derived from the diagram.

It is possible to calculate the value of the over-all theoretical  $C_z$  for this wing.  $K$  being the proportionality constant of the lift coefficient  $C_z$  determined from the tracing of each profile for infinite aspect ratio, the total value of  $K$  for the whole wing can be obtained by the expression

$$K' \sim \frac{\sum K y \Delta x}{S}.$$

The numerator of this expression is the denominator of the previously calculated expression for  $D$ . We thus obtain  $K' = 6.96$ . This value is smaller than the particular value of  $K$  for the section of maximum relative thickness 17.7%, which is 7.175. This shows that, for equal mean relative thickness, the slope of the curve  $C_z = f(i)$  is less for a tapered wing than for a rectangular wing of constant section.

Correcting the value of  $K'$  by the coefficient of reduction for the aspect ratio 6.35 of wing 2B, we find, for an angle of attack of  $10^\circ$  (for example) above the position of zero lift

$$C_z = 6.96 \times 0.688 \sin i$$

or

$$C_z = 0.83.$$

On the experimental diagram we find 0.77, indicating a loss of about 7% in lift at all angles of attack. This loss occurred, moreover, in all the wind-tunnel tests with models.

4. Results of tests with model No. 1 A.- This wing has very nearly the same characteristics as the previously investigated model No. 2 B except that  $C_{m_0}$  is constant and equal to 5.2 throughout the whole span. The previously indicated calculations, when applied to this wing, yield the following results: total  $C_{m_0}$ , 4.55; focus at 24.7% of the chord. These values agree with the results of the wind-tunnel tests.

A comparison of the polars of these two wings (Figs. 14 and 15), whose median sections are very nearly the same, shows that the assumption of the elliptical distribution of  $C_{m_0}$  slightly diminishes the maximum value of  $C_z$ , but increases the fineness at the usual flying angles and especially at small lifts for wings of constant  $C_{m_0}$ ,  $C_x$  minimum being 1.58 for wing No. 1 A and 1.22 for wing No. 2 B. Moreover, the torsional stresses on wing No. 2 B are less than those on No. 1 A. A decreasing distribution of  $C_{m_0}$  would therefore seem to be indicated in the case of a pursuit plane. In Figures 14 and 15 the angles of attack are measured from the tangent to the lower side of the median section.

## VI. Conclusions

1. Justification of assumptions.- It is not claimed that the tests with these two models afford a complete justification of the assumptions. This would doubtless require:

- 1) The measurement of the pressures at every point and for every angle of attack;
- 2) The plotting of pressure diagrams along the span and along each section by means of the measured pressures;
- 3) Verification of the agreement of the diagrams thus obtained with the theoretical diagrams obtained by the previously described method.

Toussaint and Carafoli showed, however, that, allowing for the induced velocity, the diagrams obtained for the median section of a rectangular wing of uniform section closely resemble the theoretical diagrams. It is probable, therefore, that this is true for the two wings investigated; in which case it would only remain, for the intermediate sections, to verify the agreement of the experimental diagrams with those obtained by calculation from the section diagrams. However that may be, assuming agreement between the theoretical and experimental diagrams in the median section, the verification, in the wind tunnel, of the calculated results for two wings of different pressure distribution, affords a satisfactory practical justification of the assumptions made and consequently of the method employed.

2. Effect of tapering on the polar of a wing.— Thus far we have considered only the centering of a tapered wing and its pressure distribution in diving flight. An investigation of the effect of tapering a wing on its polar would now be opportune. Such an investigation, however, would also require a

series of systematic tests, which have not yet been undertaken. However, as regards the wings satisfying our program, we have already obtained a few results.

For wings of constant section and like aspect ratio, the maximum  $C_z$  increases and  $C_z$  decreases at every angle of attack in passing successively from a rectangular to a trapezoidal and then to an elliptical wing. This is, moreover, in agreement with the induced drag calculated for these wing types.

Wings progressively decreasing in relative thickness from the plane of symmetry toward the tip generally have maximum  $C_z$ 's slightly below those of wings of uniform section having the same aspect ratio and plan form and a relative thickness equal to the mean relative thickness of the tapered wing. The fineness of the tapered wing, however, is nearly always superior, at the usual values of  $C_z$ , to the fineness of the corresponding rectangular wing.

Tapered wings generally exhibit the following peculiarity. The flow about them is stable, even at high lifts approaching the maximum  $C_z$ . The polar of a suitably tapered wing rarely exhibits sudden lift variations in the neighborhood of maximum  $C_z$ , as often happens in the polars of rectangular wings.

The polars of the two tapered wings under discussion show that, despite their considerable mean relative thickness, their fineness is equal, for the usual lifts, and superior, for large and small lifts, to that of the best rectangular wings of con-

stant section and normal relative thickness possessing the same aspect ratio as the tapered wings under consideration.

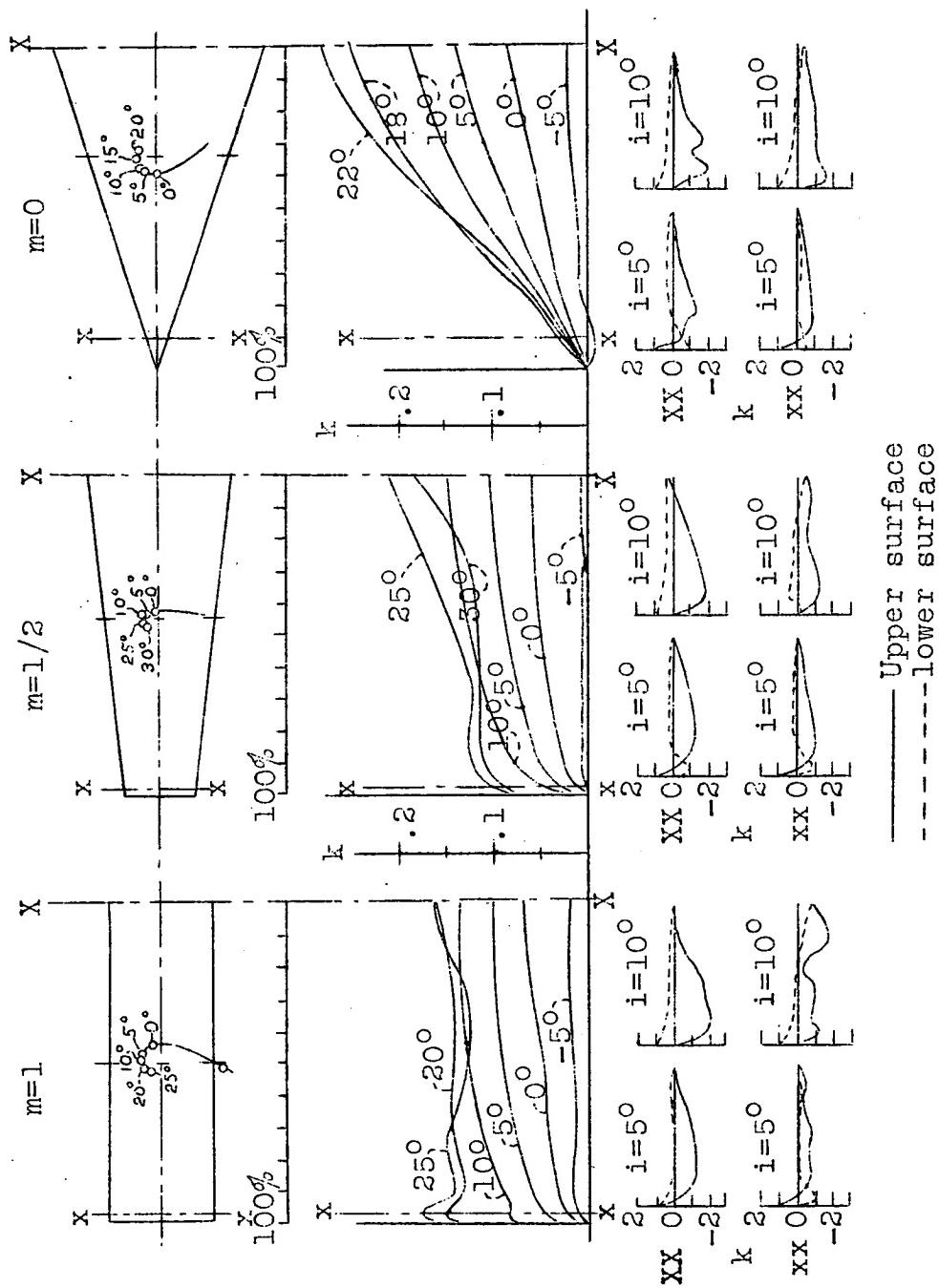
In short, I think we have justified the proposed method for calculating the aerodynamic characteristics of wings with progressively varying sections under the specified conditions. If the complex questions raised by the analysis of the local stresses on a tapered wing have not yet been solved, I trust that I have at least demonstrated the importance of the investigation of wings with progressively varying sections, notwithstanding their apparent aerodynamic equivalence with wings of current types.

Static tests, as now made, implicitly assume that  $C_{m_0}$  is constant throughout the span. It follows that, if (as is generally admitted) the pressure distribution is assumed to be uniform, the distribution of the torsional stresses must be that shown on the diagrams for  $C_{m_0}$  constant. It is possible, however, to determine the distribution of the torsional stresses in advance and then to design a wing which shall realize this distribution. The experimental verification of the adopted hypotheses would perhaps then be of a nature to warrant, for tapered wings, the application of special static-test regulations for reducing the local stresses by the use of wings of this kind.

Under such conditions, it is very probable that, with equal safety, certain forms of tapered wings, despite constructional difficulties, offer very appreciable advantages over the usual simpler forms.

In concluding, I wish to acknowledge my indebtedness to Lieutenant Colonel Alayrac, Engineer in Chief of the "Service Technique et Industriel de l'Aéronautique" for his valuable advice, and to Professor Toussaint, Director of the "Laboratoire Aérotechnique de Saint-Cyr," whose recently published works served as the basis of this analysis and to whom I am indebted for the privilege of making the necessary wind-tunnel tests.

Translation by  
National Advisory Committee  
for Aeronautics.



Figs.1,2,3, Left to right, taken from N.A.C.A. Technical Report No.150.



Fig.4 Wing sections with theoretical profiles arranged to satisfy the hypotheses mentioned in the text.

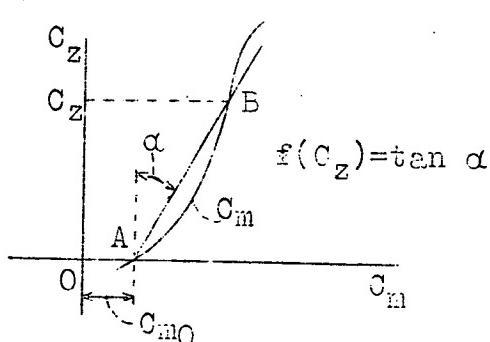


Fig.5

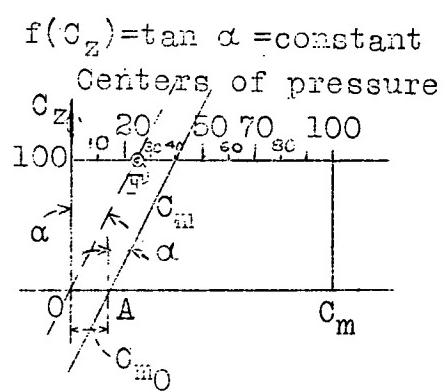


Fig.6

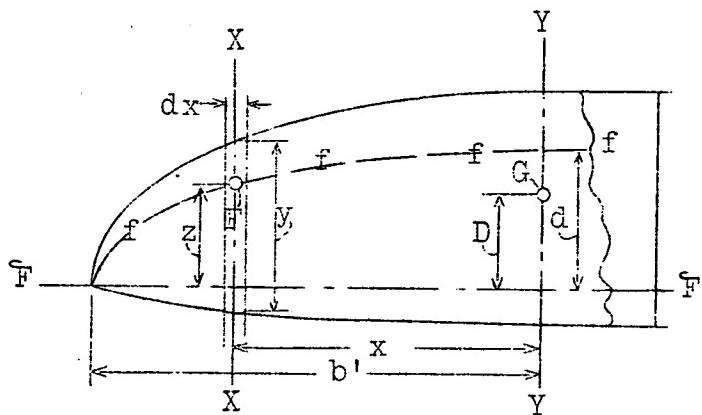


Fig.7

100

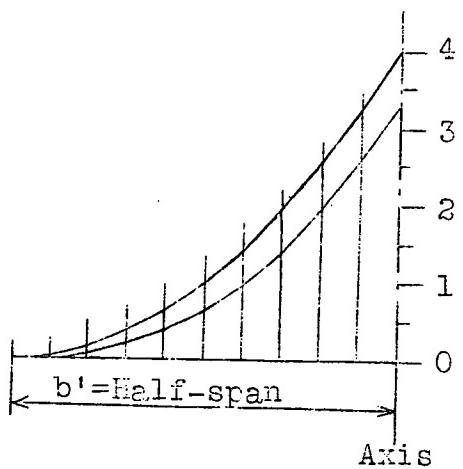


Fig.8

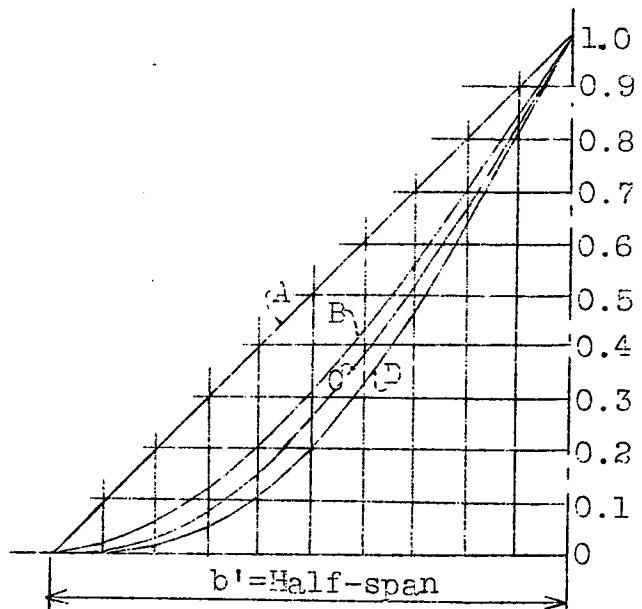


Fig.9

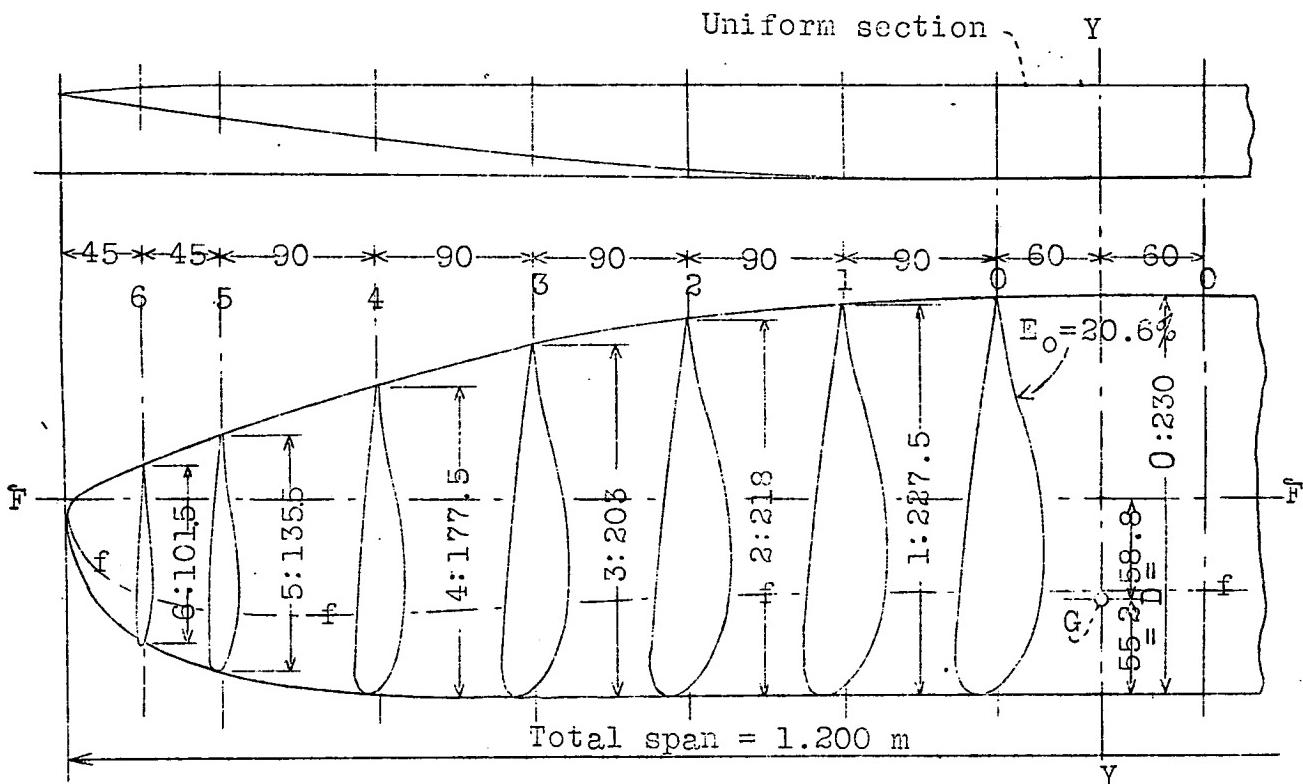


Fig.10 Wing No.2B

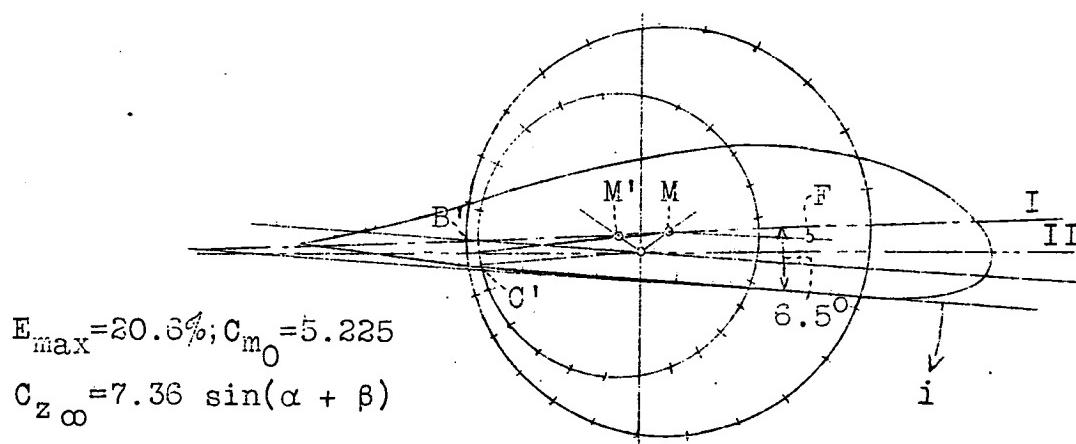


Fig.11 Wing No.2B. Section No.0

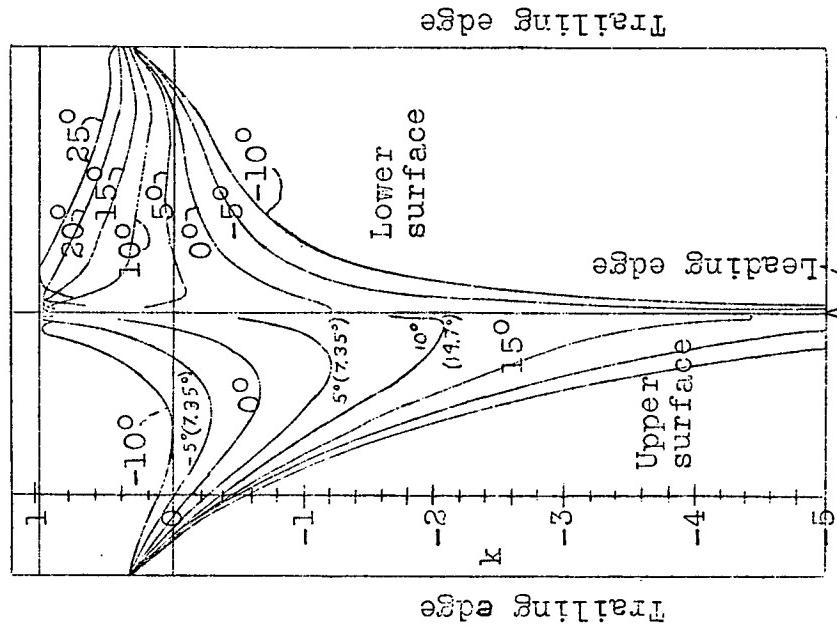


Fig.13 Section VM, No.28b.  
Wing No.2B, section No.4

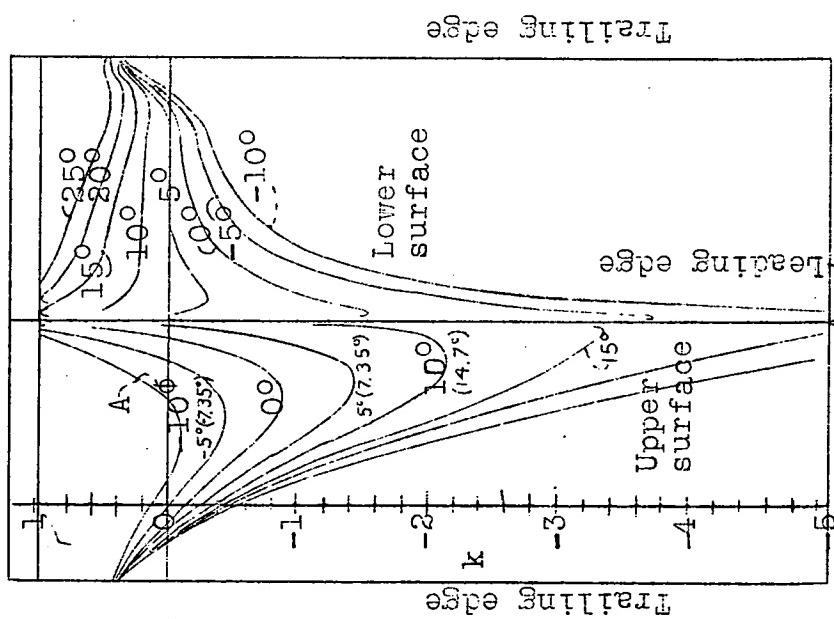
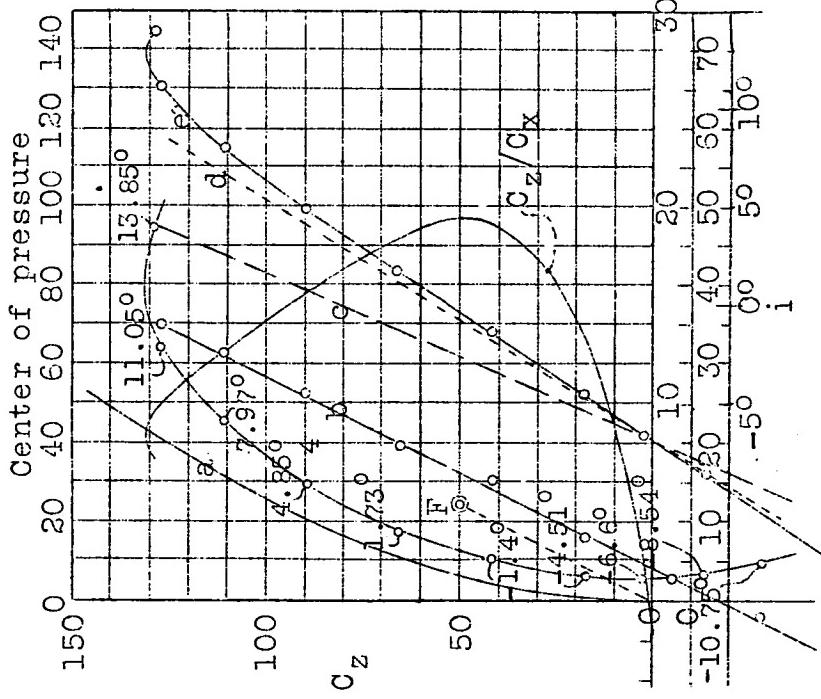


Fig.12 Section VM, No.32b.  
Wing No.2B, section No.0

a, Aspect ratio, 6.35  
b,  $C_m$  {theoretical and experimental)



c,  $C_z \infty$  (theoretical). d,  $C_z$  6.35 (theoretical)  
 e,  $C_z$  (experimental)

Fig. 14 Polar of wing No. 2B

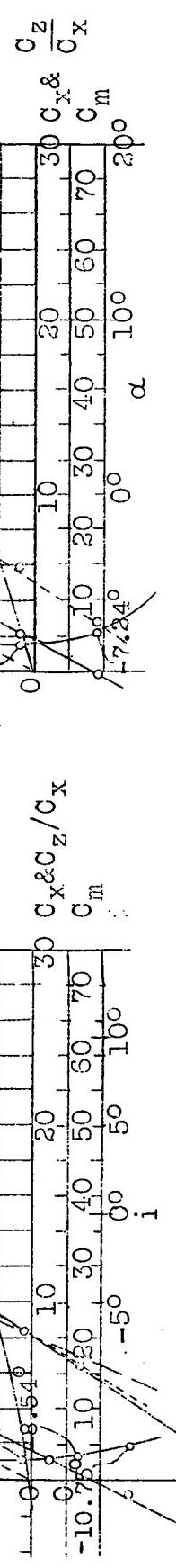
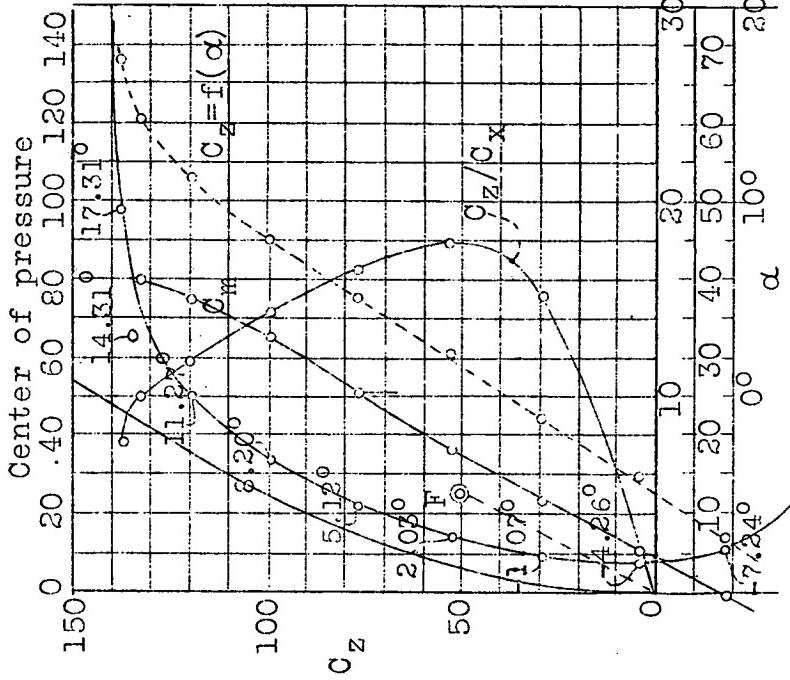


Fig. 15 Polar of wing No. 1A